Name————————————, ID ——————

MTH 330, Fundamental concepts of geometry, Fall 2014

Ayman Badawi

QUESTION 1. Draw a circle with radius 4cm, say C, centered at a point, say O. Let Q be a point inside C such that $|OQ| = 2cm$. What is the smallest radius of the circle M, where M is orthogonal (perpendicular) to C and it passes through Q?

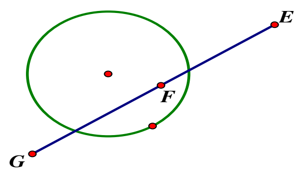
QUESTION 2. Let C and Q as in the previous question. Convince me that there is a circle D with radius $\sqrt{10}$ such that D is orthogonal to C and it passes through Q. Show the steps that you will follow in order to construct such D, you may use marked ruler.

QUESTION 3. Draw a circle with radius 6 cm, say C. Let F and W be points on the circle C such that FW is not a diameter of C. Now consider the line FW. Construct the inversion of the line FW with respect to C. You are allowed to use a marked ruler.

QUESTION 4. Let C be a circle centered at O and with radius 5cm. Let A, B be points on C such that AB is not a diameter of C. First construct a circle, say L, passes through A, B, and O. Construct the inversion of L with respect to C.

QUESTION 5. Let C be a circle centered at O and with radius 4cm. Let A and B be points such that O, A, B are not co-linear, $|OA| = 8cm$ and $|OB| = 2cm$. Construct the inversion of the line SEGMENT AB with respect to C.

QUESTION 6. Given a circle M and a line EG, see below. Construct a circle L such that L is orthogonal to M, L



passes through F, and the line EG is a tangent line to L at F.

QUESTION 7. Let C be a circle with radius 4 centered at O. Let A be a point on C. Let B, D be points on OA such that $|OB| = 1$ and $|OD| = 2$. Construct the inversion of the line segment BD with respect to C. Then find $|inv(B)inv(D)|$.

QUESTION 8. (i) What are the types of lines in the non-Euclidean hyperbolic geometry?

(ii) One of the axioms of the hyperbolic geometry is not true in the Euclidean Geometry. What am I talking about!!!?

(iii) Let H be a circle with radius 6 centered at O. Construct a circle L with radius 4 centered at O. Let A, B be points on L such that AB is not a diameter of L. Inside H, construct the non-Euclidean triangle AOB. Find $d_H(A, B)$, $d_H(O, A)$, and $d_H(O, B)$. To calculate these non-Euclidean distances use marked ruler (give your answer to the nearest one decimal).

QUESTION 9. Let H be a hyperbolic circle with radius 4. Let B be a point on H (so B is a horizon point). Construct two parallel hyperbolic lines, say L_1 and L_2, such that L1 meets L2 at B. State briefly the steps of construction.

QUESTION 10. Let C be a circle of radius 2 cm with CENTER O, and ABC is a triangle such that $|OA| = |OB| = 4$, and $|OC| = 8$. Sketch the inversion of the triangle ABC with respect to the circle C. what is the Euclidean distance between Inv(A) and Inv(C).

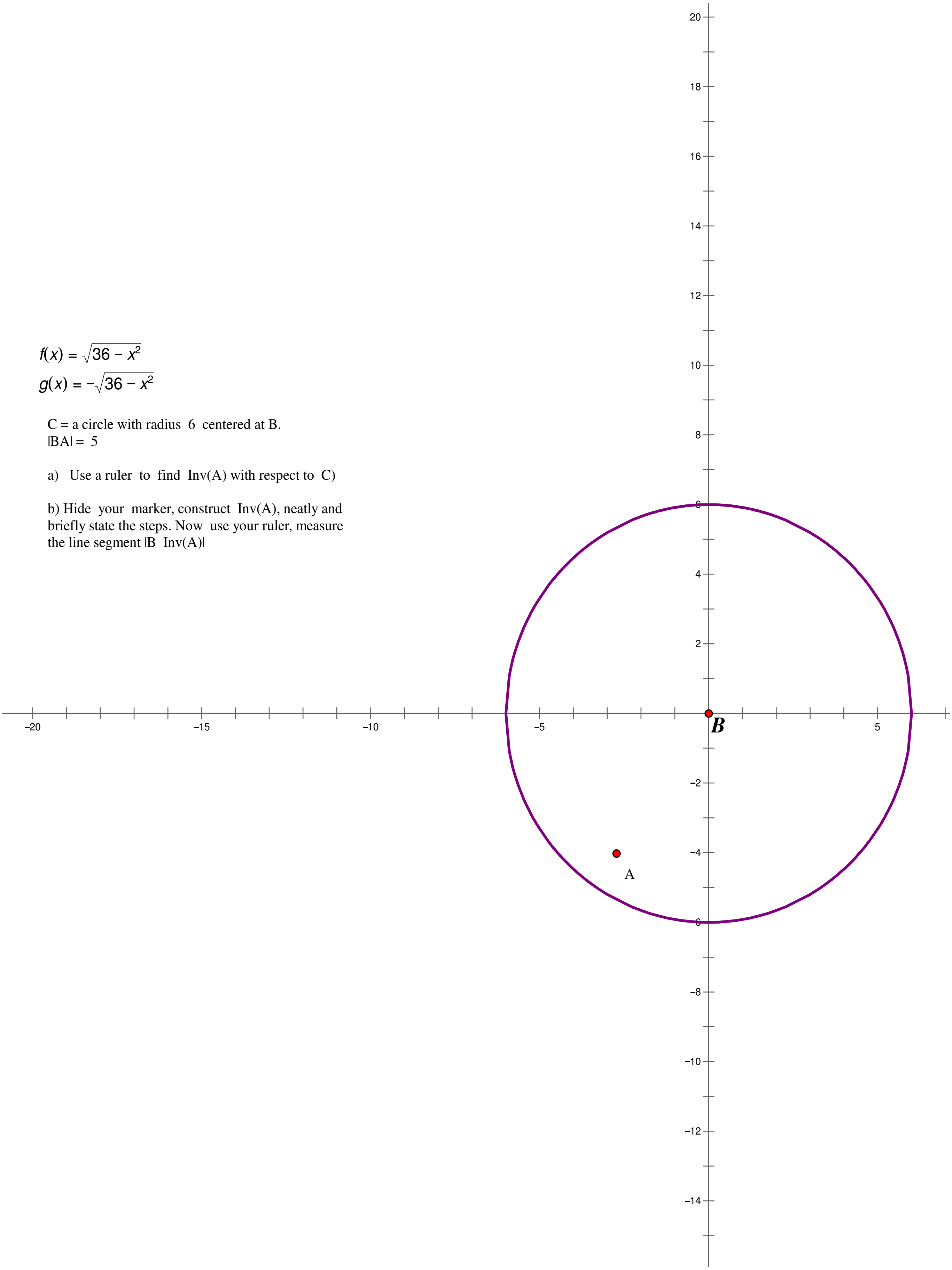
$f(x) = \sqrt{36 - x^2}$

$g(x) = -\sqrt{36 - x^2}$

C = a circle with radius 6 centered at B.
|BA| = 5

a) Use a ruler to find Inv(A) with respect to C)

b) Hide your marker, construct Inv(A), neatly and briefly state the steps. Now use your ruler, measure the line segment |B Inv(A)|



QUESTION 12. (10 points). Consider

$f(x) = \sqrt{64 - x^2}$

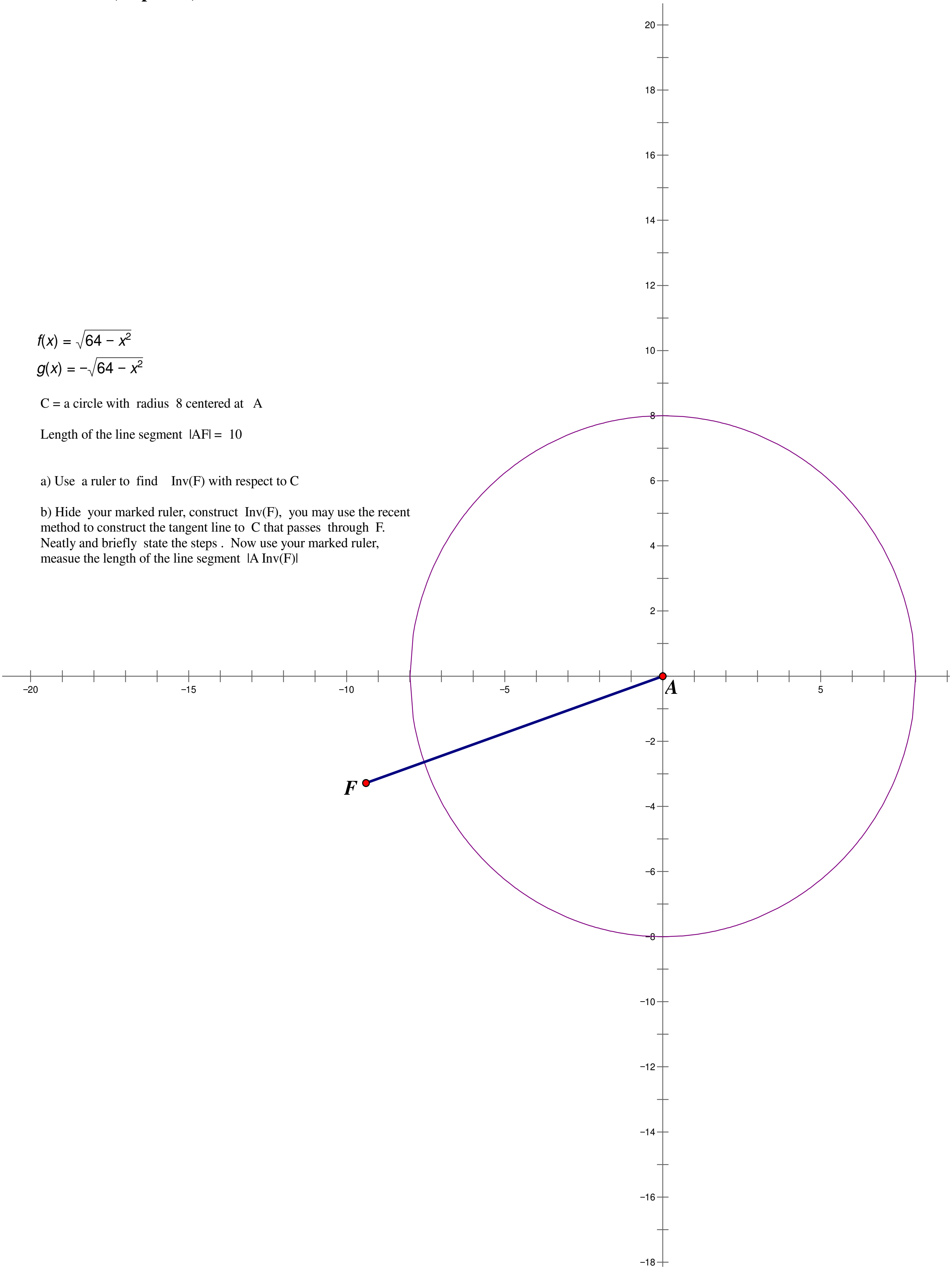
$g(x) = -\sqrt{64 - x^2}$

C = a circle with radius 8 centered at A

Length of the line segment |AF| = 10

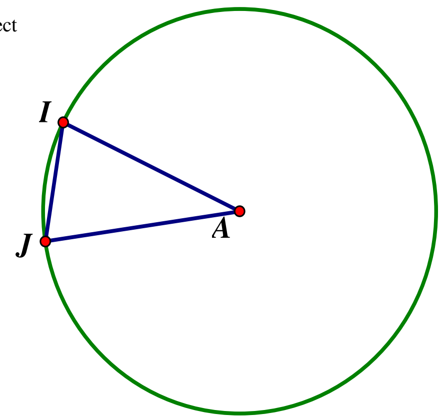
a) Use a ruler to find Inv(F) with respect to C

b) Hide your marked ruler, construct Inv(F), you may use the recent method to construct the tangent line to C that passes through F. Neatly and briefly state the steps . Now use your marked ruler, measue the length of the line segment |A Inv(F)|

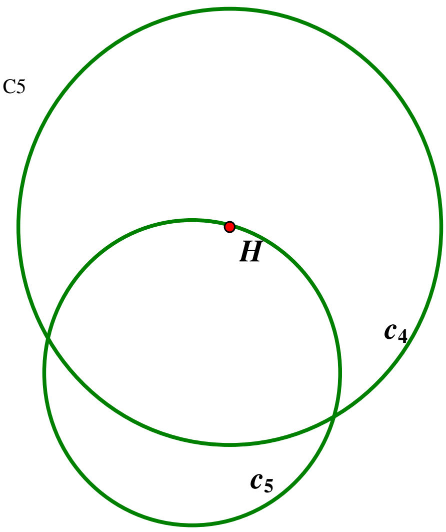


QUESTION 13. (10 points). Consider

C is a circle centered at A. Find the general shape of the inversion of the triangle AIJ with respect to C. You dont need to do the actual (exact) inversion,



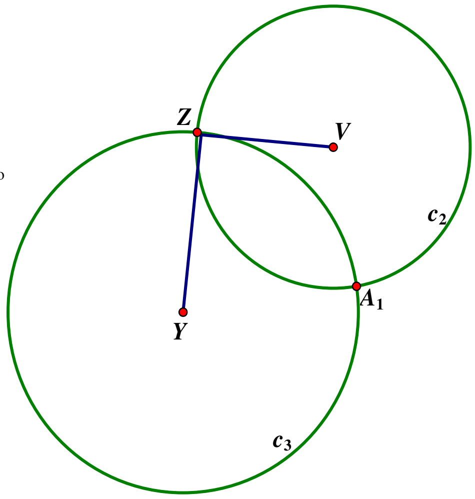
C4 is a circle centered at H. Find the exact (the actual) inversion of C5 with respect to C4.



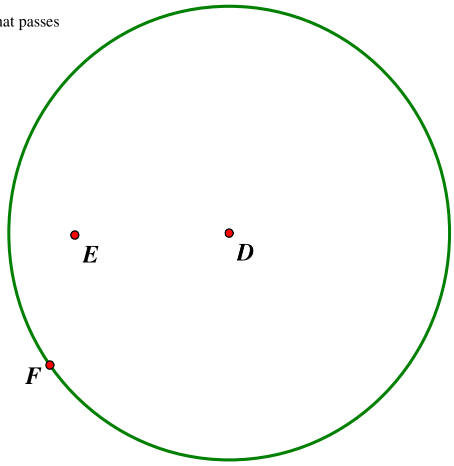
QUESTION 14. (20 points). Consider

Given ZV is perpendicular to ZY at Z. What is the inversion of the circle C3 with respect to C2? explain

What is the inversion of the arc ZA1 of the circle C3 that is inside C2 with respect to C2?



Given a circle C centered at D. State neatly and briefly the steps that you would follow in order to construct a circle M that passes through E and F such that M is orthogonal to C.



For the non-Euclidean hyperbolic geometry, answer the following::

1) Sum of the interior angles of any triangle is always ....

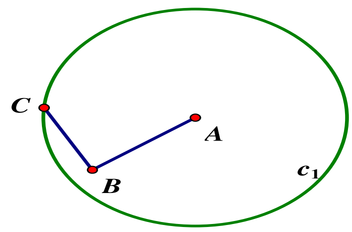
2) If Q is a point not on a line L, how many lines are there passing through Q and parallel to L?

3) If Q is a real point and B is a horizon point, then what is the maximum number of lines that are passing through Q and B?

QUESTION 15. (15 points). Consider

C1 is a circle with radius 5 centered at A, CB is perpendicular to AB at B and assume |CB| = 3.

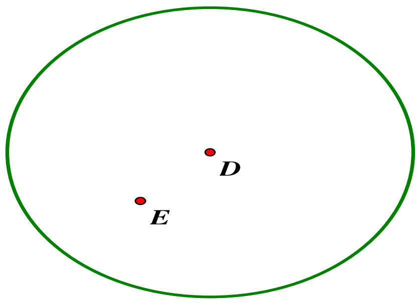
1) Find the length of the line segment AInv(B), i.e., find |AInv(B)|.



C is a circle with radius 4 centered at D. |DE| = 2. Let L be a circle passes through E and orthogonal to C.

1) What is the smallest radius of L?

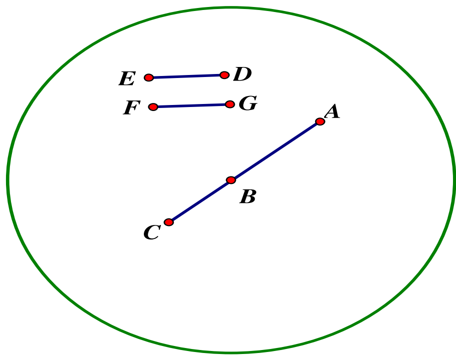
2) Can we construct such L with radius sqrt{13}? If yes construct such L with radius EXACTLY sqr{13}



QUESTION 16. (15 points). Consider

Given the Hyperbolic circle H with radius 6 centered at B. Given C, B, A lie on the same line segment AC, d(A, B) = 4, d(C, B) = 2.

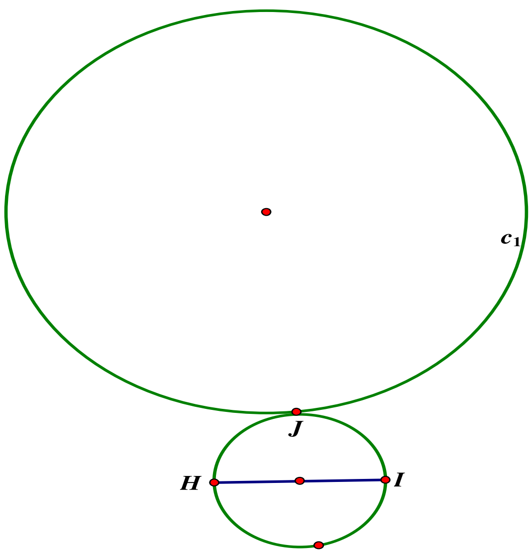
1) Find the hyperbolic distance between A and C, i.e. find d_h(A, C)



2) Given d(E, D) = d(F, G) (see picture). Can we conclude that d_h(E, D) = d_h(F, G)? d_h(E, D) < d_h(F, G)? d_h(E, D) > d_h(F, G)? briefly Explain your conclusion.

Find the inversion of HIJ with respect to C1. Just draw the general shape of the inversion (it need not be exact).

Note that HIJ consists of the line segment HI and the upperhalf ARC.



Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com